

Holographic vortices in the presence of dark matter sector

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ABSTRACT: The *dark matter* seem to be an inevitable ingredient of the total matter configuration in the Universe and the knowledge how the *dark matter* affects the properties of superconductors is of vital importance for the experiments aimed at its direct detection. The homogeneous magnetic field acting perpendicularly to the surface of (2+1) dimensional s-wave holographic superconductor in the theory with *dark matter* sector has been modeled by the additional $U(1)$ -gauge field representing dark matter and coupled to the Maxwell one. As expected the free energy for the vortex configuration turns out to be negative. Importantly its value is lower in the presence of *dark matter* sector. This feature can explain why in the Early Universe first the web of *dark matter* appeared and next on these gratings the ordinary matter forming cluster of galaxies has formed.

KEYWORDS: Gauge-gravity correspondence, Holography and condensed matter physics (AdS/CMT), Black Holes

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1 Introduction

Proposed as the equivalence between type IIB superstring theory on $AdS_5 \times S^5$ spacetime and $\mathcal{N} = 4$ $SU(N)$ supersymmetric Yang Mills theory on $(3 + 1)$ -dimensional boundary [1]-[3] and generalized to the other gravitational backgrounds [4], the gauge/gravity duality helps us to understand the physics of strongly correlated systems.

The main feature of the string theories is that gravity emerges from them in a natural way. In this context the string theories dominated by the classic gravity configurations provide a basis for the weak-strong duality [5]. In technical terms [6] it means that the partition function of the field theory $Z[A(\mathbf{x}, t)]$, where $A(\mathbf{x}, t)$ is the source field that couples to the currents $\mathbf{j}(\mathbf{x}, t)$, equals to that of the gravity dual $Z[A(\mathbf{x}, r, t)]$ with the boundary condition $\lim_{r \rightarrow \infty} A(\mathbf{x}, r, t) = A(\mathbf{x}, t)$. The latter partition function is prevailed by the classical configurations and thus given by the exponential of the classical action of the aforementioned configurations.

The AdS/CFT correspondence has been successfully applied for the description of the superconducting phase transition of the single s-wave superconductor [7], as well as, the other symmetries [8]-[26]. The effects of the higher order curvatures [27]-[32] of the gravity background and non-linearities in the electrodynamic theory [33]-[36] have been widely studied.

The aforementioned studies were also extended by the modification of gravity theory by considering the five-dimensional AdS solitonic metric [37], which enables to construct a model of the holographic insulator/superconductor phase transition at zero temperature [38]. The AdS soliton line element dual to a confined field theory with a mass gap, imitates an insulator phase [39].

The effects of magnetic field (which will be crucial in our further studies) on holographic superconductors were also intensively studied. It was revealed that by adding magnetic charge to a black hole, the holographic superconductor could be immersed in magnetic field [40]. The studies of the problem in question were also conducted in Gauss Bonnet gravity and in Weyl-corrected and non-linear electrodynamics [41]-[44]. Holographic vortices and droplets influenced by magnetic field were elaborated in [45]-[46], while the vector condensation generated by this factor was studied in [19, 47]. On the other hand, the magnetic penetration depth was computed in [7], where it was envisaged that the holographic superconductors are of type II. In order to find the superconducting coherence length and magnetic penetration depth [48] one perturbs the AdS-Schwarzschild system mimicking s-wave superconductor near the critical temperature. These works triggered the investigations in p-wave as well as d-wave holographic superconductors [49]-[50]. It has been found that the superconducting coherence length was proportional to $(1 - T/T_c)^{-1/2}$ near the critical temperature. Similar dependence of the magnetic penetration depth $\lambda \propto (1 - T/T_c)^{-1/2}$ has been observed. These results agree with the standard Ginzburg-Landau theory.

In [51] the interesting case of the Abelian Chern-Simons Higgs model in $(2 + 1)$ -dimensions was elaborated. From this point of view the higher derivative corrections to the Abelian gauge sector in the bulk were studied [52]. The vortex lattice solution in a holographic superconductors constructed from a charged scalar condensate was elaborated in [53]. The perturbative solution near the second-order phase transition represents the holographic Abrikosov lattice. On the other hand, the holographic vortex-flow solution was given in [54].

The other crucial problem is the question about the possible matter configuration in the AdS spacetime. This problem was considered in the case of strictly stationary Einstein-Maxwell AdS spacetime [55] and its generalization to the low-energy limit of the heterotic string theory with arbitrary number of $U(1)$ -gauge fields [56]. It was concluded that the spacetimes in question can not allow for the existence of nontrivial configurations of complex scalar or form fields.

The origin of cosmic structures can be understood in terms of evolution of matter perturbations arising after inflationary period. The *dark matter* constitutes the key ingredient in the aforementioned processes, as a potential wells where the ordinary matter accrete. If *dark matter* appears in the form of particles coupled to ordinary matter then a certain level of emitted radiation is expected. The non-gravitational signals are anticipated from interaction of dark matter with ambient medium, annihilation or decay by a direct emission or through particle production. The non-gravitational signals of it should be in principle proportional to the density of clumps of dark matter. Recently, a new method to constrain the process of elastic scattering between dark matter and the Standard Model particles in the Early Universe was proposed [57, 58].

Dark matter may also leave its footprints during collapse of neutron stars and in the first generation of stars [59]-[61]. On the other hand, the behavior of *dark matter* and dark energy collapse is studied numerically from the point of view of possible features of black hole and wormhole formations [62]-[63]. This is an important problems in studies of the black hole growth in the Early Universe.

The dark matter non-gravitational interactions, having implications for the particle physics beyond the Standard Model, triggered search for high-energy photons produced by annihilation which in turn causes the resurgence in inspection of gamma-rays emission from the dwarf galaxies [64]. This problem is also interesting in the context of planing varieties of gamma-rays telescopes in the range of energy from 1 to 100MeV [65]. Investigations of dilatonic like coupling to photons which can be caused by ultralight dark matter was reported in [66]. This process can induce oscillations in the fine-structure constant.

The *dark matter* sector model may have a strong support by some astrophysical data such as observation of 511 keV gamma rays [67] experiments detecting the electron positron excess in galaxy [68, 69], and possible explanation of muon anomalous magnetic moment[70]. Recent astronomical observations reveal that collisions among galaxy clusters provide new tests of non-gravitational forces acting on dark matter [71, 72]. The experiments in question may disfavor some extensions of the Standard Model.

The idea that *dark matter* sector is charged under a new $U(1)$ -gauge group and is coupled to the ordinary Maxwell field gained experimental verifications. The low-energy $e^+ e^-$ colliders offer the environment to probe low-energy mass *dark sector* [73]. The data from BABAR detector were revealed bounded with a search for dark photon production in the range of mass $0.02 < m < 10.2GeV$, but no significant signal has been observed. The other experiments are planned to cover the energy region $15 \leq m \leq 30MeV$.

It is thus obvious that study of *dark matter* and its observational consequences is both timely and of crucial importance. In this paper we add a novel aspect to it by analyzing the behavior of holographic superconductors.

We shall try to answer the question how a *dark matter* sector modifies the ordinary phase transitions known from the pervious studies and what the role of the dark matter coupling constant α , binding dark matter fields with ordinary Maxwell gauge field is. We investigate analytically the perturbation of the dual gravity theory to reveal the superconducting coherence length and magnetic penetration depth close to the superconducting phase transition point. The main aim of this paper is to study the influence of the *dark matter* sector on the holographic vortex. In the previous works [74]-[78], various aspects of phase transitions in s-wave and p-wave holographic superconductor theory with ordinary matter sector coupled to another $U(1)$ -gauge field which describes *dark matter* have been elaborated. One may expect that *dark matter* authorizes a part of a larger particle sector interacting with the visible matter and not completely decoupled [79]-[85].

The case of homogeneous magnetic field acting perpendicularly to the surface of s-wave holographic superconductor in the theory with *dark matter* sector has been considered. *Dark matter* has been modeled by the additional $U(1)$ -gauge field coupled to Maxwell one and $(2 + 1)$ -dimensional vortices have been studied in the considered theory. Free energy for the vortex configuration turns out to be negative and it is less for the configuration

with *dark matter* sector than for ordinary matter case. One concludes that the dark matter sector vortex configurations are more stable than trivial configuration where there is no charge scalar condensate. On the other hand, because of the fact that *dark matter* vortices build less free energy configuration than ordinary matter ones, this fact can explain why in the Early Universe first form the web of *dark matter* and next on these tendrils the ordinary matter condenses forming cluster of galaxies.

The paper is organized as follows. In section 2 we describe the main features of the theory with *dark matter* section which is mimicked by the coupling between ordinary Maxwell and $U(1)$ -gauge fields. In section 3, to the leading order, we find perturbatively solutions of the equations of motion in the underlying theory. Section 4 is devoted to free energy for the considered system which determines thermodynamics stability of the *dark matter* sector vortex. Section 5 is connected with calculations of the superconducting coherence length and magnetic penetration depth which depend on the α -coupling constant of the *dark matter* part of the theory. In section 6 we find the analogy of the London equation for the holographic vortex in the theory in question and calculate the superfluid density and magnetic penetration depth. Both quantities are affected by the influence of the *dark matter* section of the considered theory.

2 Superconductor with dark matter sector in AdS/CFT theory

The gravitational action in (3+1) dimensions is taken in the form

$$S_g = \int \sqrt{-g} d^4x \left(R - 2\Lambda \right), \quad (2.1)$$

where $\Lambda = -3/L^2$ stands for the cosmological constant, while L is the radius of the AdS spacetime. We shall examine the Abelian-Higgs sector coupled to the second $U(1)$ -gauge field which mimics the *dark matter* sector of the underlying theory [83]. The adequate action incorporating *dark matter* is provided by

$$S_m = \int \sqrt{-g} d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - [\nabla_\mu \psi - iq A_\mu \psi]^\dagger [\nabla^\mu \psi - iq A^\mu \psi] - m^2 |\psi|^2 \right. \\ \left. - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\alpha}{4} F_{\mu\nu} B^{\mu\nu} \right), \quad (2.2)$$

where $F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$ stands for the ordinary Maxwell field strength tensor, while the second $U(1)$ -gauge field $B_{\mu\nu}$ is given by $B_{\mu\nu} = 2\nabla_{[\mu} B_{\nu]}$. Moreover, m , q represent, respectively a mass and a charge related to the scalar field ψ . α is a coupling constant between $U(1)$ fields. Varying the action with respect to the considered fields one obtains the equations of motion which can be written in the form as

$$\left(\nabla_\mu - i q A_\mu \right) \left(\nabla^\mu - i q A^\mu \right) \psi - m^2 \psi = 0, \quad (2.3)$$

$$\tilde{\alpha} \nabla_\mu F^{\mu\nu} = j^\nu, \quad (2.4)$$

where the current j^ν yields

$$j^\nu = i \left[\psi^\dagger \left(\nabla^\nu - i q A^\nu \right) \psi - \psi \left(\nabla^\nu + i q A^\nu \right) \psi^\dagger \right] = 0, \quad (2.5)$$

and $\tilde{\alpha} = 1 - \alpha^2/4$. In the above relation we have used the equation determining $B_{\mu\nu}$ field

$$\nabla_\mu B^{\mu\nu} + \frac{\alpha}{2} \nabla_\mu F^{\mu\nu} = 0, \quad (2.6)$$

in order to eliminate this field and obtain the relation (2.4).

The presence of the *dark matter* sector is marked by the appearance of $\tilde{\alpha}$ in the equations of motion. From this point of view the holographic *s* and *p*-wave superconductor phase transitions were previously studied [74–76] in the presence of dark matter. It was revealed that there were some imprints of *dark matter* sector on holographic phase transitions. In particular it has been found [76] that in the probe limit the prefactor Γ between the value of the condensing operator in the relation $\langle \mathcal{O} \rangle = \Gamma \sqrt{1 - T/T_c}$ does depend on α . $\Gamma \propto \sqrt{\tilde{\alpha}}$. Neither the superconducting transition temperature nor critical value of the chemical potential for insulator to metal transition depend on α . Much more severe dependence on the *dark matter* coupling has been observed in the backreacting theory [74], where the transition temperature depends on α . The p-wave superconductors described by the SU(2) Yang-Mills theory features *inter alia* the increase [77] of the transition temperature with increase of the coupling α . On the other hand, the critical chemical potential μ_c for the quantum phase transition between insulator and a metal is a decreasing function of α .

In the theory with the action given by equations (2.1) and (2.2) the gravitational background will be provided by the four-dimensional AdS-Schwarzschild black hole background of the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2} (dx^2 + dy^2), \quad (2.7)$$

where the metric function reads

$$f(r) = \frac{r^2}{L^2} \left(1 - \frac{r_+^3}{r^3} \right). \quad (2.8)$$

L is the radius of the AdS spacetime and r_+ is the horizon of the black hole. The Hawking temperature of the black hole is found to be

$$T = \frac{3r_+}{4\pi L^2}. \quad (2.9)$$

In what follows, without loss of generality, we shall set the spacetime radius L and the charge related to the scalar field equal to 1. One also changes the variables replacing the space coordinate r by $u = r_+/r$. This replacement results in the line element of the form

$$ds^2 = -f(u)dt^2 + \frac{r_+^2}{u^4 f(u)}du^2 + \frac{r_+^2}{u^2} (dx^2 + dy^2), \quad (2.10)$$

where $f(u) = r_+^2/u^2 (1 - u^3)$.

We are interested in the properties of superconductors subject to the external magnetic field. As it is well known from experiments on superconducting systems and in agreement with Ginzburg-Landau theory, type I superconductors are characterized by the single value of the critical magnetic field B_c , beyond which the superconductivity is destroyed. Such systems are characterized by the penetration depth λ smaller than the superconducting

coherence length ξ . The existence of the border between normal metal and superconductor increases its energy and makes the system thermodynamically unstable. On the other hand, in the opposite limit of $\lambda/\xi \gg 1$ (type II superconductors) the existence of the border decreases the energy and allows the magnetic field to penetrate into the metal in the form of vortices. The process starts at the "lower magnetic field" B_{c1} and continues until "upper magnetic field" B_{c2} , when the vortices are so dense that they overlap and the system undergoes superconductor to normal metal transition. The vortices are normal systems at their cores. We do not consider here the dynamics of the gauge field. It is important in analysis of various properties of superconductors [86, 87] in particular the Meissner effect.

One should remark that in the considered theory there are in principle two ways of choosing the magnetic field. One is the ordinary magnetic field introduced by the appropriate component of the $U(1)$ -gauge Maxwell field, A_y . The other, is to choose the magnetic field of *dark matter* sector and then by the equation of motion (2.6) receive the Maxwell component of A_y which enters the relation (2.4). Choosing the second possibility we assume constant dark matter magnetic field \hat{B}

$$B_y = \hat{B} x. \quad (2.11)$$

Thus, having in mind equation (2.6), we obtain

$$A_y = \frac{2}{\alpha} \left(c_1 - \hat{B} \right) x + c_2, \quad (2.12)$$

where $c_{1,2}$ are arbitrary constants. In our further considerations we shall keep the general form of A_y and in the resulting conclusion one specifies the influence of the two aforementioned possibilities of choosing magnetic field.

To proceed further we shall solve the equations of motion perturbatively in the probe limit. Namely, we define the deviation parameter $\epsilon_H = (B_{c2} - B)/B_{c2}$, with the property that $|\epsilon_H| \ll 1$, and expand gauge fields $A_\mu(\mathbf{x}, u)$, the scalar field $\psi(\mathbf{x}, u)$ and the current $j_\mu(\mathbf{x}, u)$ in series of ϵ_H

$$A_\mu(\mathbf{x}, u) = A_\mu^{(0)}(\mathbf{x}, u) + \epsilon_H A_\mu^{(1)}(\mathbf{x}, u) + \dots, \quad (2.13)$$

$$\psi(\mathbf{x}, u) = \epsilon_H^{\frac{1}{2}} \psi_1(\mathbf{x}, u) + \epsilon_H^{\frac{3}{2}} \psi_2(\mathbf{x}, u) + \dots, \quad (2.14)$$

$$j_\mu(\mathbf{x}, u) = \epsilon_H j_\mu^{(1)}(\mathbf{x}, u) + \epsilon_H^2 j_\mu^{(2)}(\mathbf{x}, u) + \dots, \quad (2.15)$$

where $\mathbf{x} = (\mathbf{x}, \mathbf{y})$. The above chosen expansions are analogous to that used previously [48–54]. The different powers of ϵ_H are related to the equations of motion for the scalar field ψ and A_t as discussed in [48]. They envisage the fact that the scalar field ψ acts as an order parameter and vanishes at the critical value of the magnetic field in the mean field like manner. In what follows we solve and analyze the solutions of the equations (2.3) and (2.4), for A_μ and ψ , up to the leading order.

3 Leading orders of the equations of motion

3.1 Zeroth order

Substituting the relations (2.13)-(2.15) into (2.3) and (2.4), we obtain the zeroth order equations of motion

$$\tilde{\alpha} \nabla_\nu F^{\nu\mu(0)} = 0. \quad (3.1)$$

In accordance with the assumed dependence of the fields in question on the parameter ϵ_H , in zeroth order only the components of the gauge fields have non-zero values. In order to find the solutions we consider the ansatz as follows:

$$A_\mu^{(0)}(\mathbf{x}, u) = (A_t(\mathbf{x}, u), A_u(\mathbf{x}, u), A_x(\mathbf{x}, u), A_y(\mathbf{x}, u)) = (\varphi(u), 0, 0, A_y^{(0)}). \quad (3.2)$$

The zeroth order solution generates the chemical potential μ and the critical magnetic field B_{c2} as given by the relations

$$A_t^{(0)} = \mu (1 - u), \quad A_u^{(0)}, \quad A_x^{(0)} = 0, \quad A_y^{(0)} = B_{c2} x. \quad (3.3)$$

The solution agrees with the fact that at $B = B_{c2}$ the superconducting order parameter and the currents vanish.

3.2 Next to zeroth order

In this subsection one derives equations of motion for the scalar field ψ , the Maxwell gauge potentials A_μ and the current. As it follows from the assumed expansion, the non-zero value of the condensate ψ_1 is a coefficient in front of the $\sqrt{\epsilon_H}$ while the current being proportional to "square" of the scalar field appears in the order ϵ_H . This provides a posteriori justification of the expansion (2.13)-(2.15).

3.2.1 Scalar field equation

Up to the $\sqrt{\epsilon_H}$ -order the equation of motion for the scalar field can be written as

$$\partial_u \left[f(u) \partial_u \psi_1 \right] + \frac{1}{u^2} \left(\Delta - 2i A_y^{(0)} \partial_y - (A_y^{(0)})^2 \right) \psi_1 + \frac{r_+^2 \varphi^2(u)}{f(u) u^4} \psi_1 - \frac{m^2 r_+^2}{u^4} \psi_1 = 0. \quad (3.4)$$

In order to solve the above equations one separates the variables

$$\psi_1(u, \mathbf{x}) = \rho_0(u) H(\mathbf{x}) = \rho_0(u) e^{ik_y y} X(x). \quad (3.5)$$

Inserting the above ansatz for $\psi_1(u, \mathbf{x})$ we arrive at the set of equations provided by

$$\rho_0''(u) + \frac{f'(u)}{f(u)} \rho_0'(u) + \frac{r_+^2 \varphi^2(u)}{f(u) u^4} \rho_0(u) - \frac{m^2 r_+^2}{u^4} \rho_0(u) = \frac{\rho_0(u)}{\zeta^2 u^2 f(u)}, \quad (3.6)$$

$$-X''(x) + (A_y^{(0)})^2 \left(x - \frac{k_y}{A_y^{(0)}} \right)^2 X(x) = \frac{X(x)}{\zeta^2}, \quad (3.7)$$

where by ζ^2 we denoted a separation constant.

3.2.2 Solution for A_μ

With the choice of gauge in the form $A_u = 0$, and assuming that the scalar field is the real one, the generalized Maxwell type equations at first order in ϵ_H are provided by

$$D^{(t)} A_t^{(1)} = 2 \frac{A_t^{(0)} |\psi_1|^2 r_+^2}{\tilde{\alpha} u^2}, \quad (3.8)$$

$$D^{(i)} A_m^{(1)} - \partial_m \left(\delta^{fk} \partial_f A_k^{(1)} \right) = \frac{j_m^{(1)} r_+^2}{\tilde{\alpha} u^2}, \quad (3.9)$$

$$\partial_u \left(\delta^{ac} \partial_a A_c^{(1)} \right) = 0, \quad (3.10)$$

where the differential operators are denoted by

$$D^{(t)} = u^2 f(u) \partial_u^2 + \Delta, \quad D^{(i)} = \partial_u \left(u^2 f(u) \partial_u \right) + \Delta. \quad (3.11)$$

By Δ we denoted the two-dimensional Laplacian, $\Delta = \partial_x^2 + \partial_y^2$. As mentioned in [53], beside the gauge transformation equaling u -component of $U(1)$ -gauge field to zero also there exist transformation of the form $A_m \rightarrow A_m - \partial_m K(\mathbf{x})$ (the so-called residual transformation). It can be seen that $\langle J_\mu \rangle \propto F_{u\mu}$ is invariant under the transformation in question. This transformation allows us to set $\delta^{ac} \partial_a A_c^{(1)} = 0$ and get the following equations instead of (3.8)-(3.10) [53]. Namely, one obtains

$$D^{(t)} A_t^{(1)} = 2 \frac{r_+^2 A_t^{(0)}}{\tilde{\alpha} u^2} \rho_0^2(u) \sigma(\mathbf{x}), \quad (3.12)$$

$$D^{(i)} A_x^{(1)} = \frac{r_+^2}{\tilde{\alpha} u^2} \rho_0^2(u) \epsilon_x^y \partial_y \sigma(\mathbf{x}), \quad (3.13)$$

$$D^{(i)} A_y^{(1)} = \frac{r_+^2}{\tilde{\alpha} u^2} \rho_0^2(u) \epsilon_y^x \partial_x \sigma(\mathbf{x}), \quad (3.14)$$

where $\epsilon_{ab} = \epsilon^a_b$ is the two-dimensional totally antisymmetric symbol with the properties $\epsilon_{xy} = -\epsilon_{yx} = 1$. In the above relations we have introduced $\sigma(\mathbf{x}) = |H(\mathbf{x})|^2$ represents the density of the condensate per unit volume around the point \mathbf{x} .

The boundary conditions for the equation (3.8) are given by demanding that $A_t^{(1)}$ is equal zero at $u = 0$ and for $u = 1$. For the equation (3.9) we impose that the solution is regular on the event horizon and $\epsilon_H F_{xy}^{(1)}$ calculated at the boundary of the AdS spacetime as given by the difference between magnetic and critical magnetic field $B - B_{c2}$.

The formal solutions of the above relations can be cast in the form as

$$A_t^{(1)} = -2 \frac{r_+^2}{\tilde{\alpha}} \int_0^1 du' \frac{\rho_0^2(u')}{u'^2} A_t^{(0)} \int d\mathbf{x}' G_B(u, u' | \mathbf{x} - \mathbf{x}') \sigma(\mathbf{x}'), \quad (3.15)$$

$$A_m^{(1)} = a_m(\mathbf{x}) - \frac{r_+^2}{\tilde{\alpha}} \epsilon_m^j \int_0^1 du' \frac{\rho_0^2(u')}{u'^2} \int d\mathbf{x}' G_s(u, u' | \mathbf{x} - \mathbf{x}') \partial_j \sigma(\mathbf{x}'), \quad (3.16)$$

where the Green functions connected with the time and spatial components of the Maxwell gauge potentials imply

$$D^{(t)} G_B(u, u' | \mathbf{x} - \mathbf{x}') = -\delta(u - u') \delta(\mathbf{x} - \mathbf{x}'), \quad (3.17)$$

$$D^{(i)} G_s(u, u' | \mathbf{x} - \mathbf{x}') = -\delta(u - u') \delta(\mathbf{x} - \mathbf{x}'), \quad (3.18)$$

with the AdS boundary conditions provided by the following:

$$G_B(u, u' | \mathbf{x} - \mathbf{x}')|_{u=0} = G_B(u, u' | \mathbf{x} - \mathbf{x}')|_{u=1} = 0, \quad (3.19)$$

$$G_s(u, u' | \mathbf{x} - \mathbf{x}')|_{u=0} = G_s(u, u' | \mathbf{x} - \mathbf{x}')|_{u=1} = 0. \quad (3.20)$$

3.2.3 Current

The AdS prescription enables us to find the $U(1)$ -gauge fields current

$$\langle J^\beta \rangle = \frac{\delta S_{on-shell}}{\delta A_\beta} \Big|_{u \rightarrow 0} = \left(F^{\beta u} + \frac{\alpha}{2} B^{\beta u} \right) \Big|_{u \rightarrow 0}. \quad (3.21)$$

Using equation (3.16) for $A_j^{(1)}$ and $F_{ju}^{(1)} = -\partial_u A_j^{(1)}$ we arrive at the following relation for the spatial components of the current in question

$$\langle J_i \rangle^{DM} = \left(F_{iu} + \frac{\alpha}{2} B_{iu} \right) \Big|_{u \rightarrow 0}. \quad (3.22)$$

In order to evaluate it let us consider the u -component of the first order expansions of the gauge fields in equation of motion (2.6). It implies

$$\partial_i \left[\sqrt{-g} \left(B^{iu(1)} + \frac{\alpha}{2} F^{iu(1)} \right) \right] = 0. \quad (3.23)$$

Having in mind the line element describing our spacetime we obtain the relation for the covariant components of the gauge fields provided by

$$B_{iu}^{(1)} + \frac{\alpha}{2} F_{iu}^{(1)} = \frac{c_i r_+}{u^2 f(u)}, \quad (3.24)$$

where c_i are constants bounded with x or y components, respectively. Next, extracting $B_{iu}^{(1)}$ from the last equation and inserting it into $\langle J_i \rangle^{DM}$ we get

$$\langle J_i \rangle^{DM} = \tilde{\alpha} F_{iu}^{(1)} \Big|_{u \rightarrow 0} + \beta_i(\alpha), \quad (3.25)$$

where we set

$$\beta_i(\alpha) = \frac{\alpha c_i}{2 r_+}. \quad (3.26)$$

By virtue of the above one finally arrives at the expression for the $U(1)$ -gauge current in the theory with *dark matter* sector. Calculating $F_{iu}^{(1)}$ using equation (3.16) implies

$$\langle J_i \rangle^{DM} = \epsilon_i^m \partial_m \Theta(\mathbf{x}) \Big|_{u \rightarrow 0} + \beta_i(\alpha), \quad (3.27)$$

where we have denoted

$$\Theta(\mathbf{x}) = r_+^2 \int_0^1 du' \frac{\rho_0^2(u')}{u'^2} \partial_u \int d\mathbf{x}' G_s(u, u' | \mathbf{x} - \mathbf{x}') \sigma(\mathbf{x}') \Big|_{u \rightarrow 0}. \quad (3.28)$$

One can see that the dependence on the *dark matter* sector (dependence on $\tilde{\alpha}$) cancels in the first term and the current in the presence of the *dark matter* sector is provided by

$$\langle J_a \rangle^{DM} = \langle J_a \rangle + \beta_a(\alpha). \quad (3.29)$$

From the above relation it can be concluded that if we take an integration constants c_i equal to zero, one receives that the *dark matter* sector current is the same as in the 'ordinary' case.

4 Free energy

In this section we shall find free energy of the considered system, i.e., Maxwell *dark matter* sector vortex configuration in $(2+1)$ -dimensions. The free energy notion is important from the point of view of the thermodynamical stability of the aforementioned configuration. It turns out that in holographic approach to the physical problems the free energy for the boundary theory can be determined as the on-shell gravity action in the bulk.

In our case the on-shell action bounded with the scalar field in question vanishes identically. It is caused by the fact that the scalar field has a compact support and satisfies the boundary conditions of the form $\psi \sim c_1 u^2$, where c_1 is constant, at the AdS boundary. By virtue of the above one can restrict the attention to Maxwell and *dark matter* $U(1)$ -gauge fields.

$$S_{on-shell} = - \int_M d^4x \sqrt{-g} \left(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha}{4} F_{\mu\nu} B^{\mu\nu} \right). \quad (4.1)$$

Let us expand the above action in terms of ϵ_H

$$S_{on-shell} = S^{(0)} + \epsilon_H S^{(1)} + \epsilon_H^2 S^{(2)} + \dots \quad (4.2)$$

On the other hand, for the gauge fields one obtains

$$F_{\mu\nu}^{(i)} = \partial_\mu A_\nu^{(i)} - \partial_\nu A_\mu^{(i)}, \quad B_{\mu\nu}^{(i)} = \partial_\mu B_\nu^{(i)} - \partial_\nu B_\mu^{(i)}, \quad i = 0, 1, 2, \dots \quad (4.3)$$

The $S^{(0)}$ coefficient implies

$$- S^{(0)} = \int_{\mathcal{M}} d^4x \sqrt{-g} \left(\frac{1}{4} F_{\mu\nu}^{(0)} F^{\mu\nu(0)} + \frac{\alpha}{4} F_{\mu\nu}^{(0)} B^{\mu\nu(0)} \right). \quad (4.4)$$

The first term on the right-hand side of (4.4) accords to a trivial configuration without any scalar field, i.e., $\psi = 0$. However, the other terms in the expansion described by the relation (4.2) possess the information about states in which one has non-zero condensate.

Let us consider the $S^{(1)}$ coefficient which may be rewritten in the form as

$$\begin{aligned} - S^{(1)} &= \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{1}{2} F_{\mu\nu}^{(1)} F^{\mu\nu(0)} + \frac{\alpha}{4} \left(F_{\mu\nu}^{(1)} B^{\mu\nu(0)} + F_{\mu\nu}^{(0)} B^{\mu\nu(1)} \right) \right] \\ &= \int_{\partial\mathcal{M}} d\Sigma_u F^{u\nu(0)} A_\nu^{(1)}|_{u \rightarrow 0} + \frac{\alpha}{2} \left[\int_{\partial\mathcal{M}} d\Sigma_u \left(B^{u\nu(0)} A_\nu^{(1)} + F^{u\nu(0)} B_\nu^{(1)} \right) \right]_{u \rightarrow 0}, \end{aligned} \quad (4.5)$$

where we used the equations of motion for the zeroth order, i.e., $\nabla_\mu F^{\mu\nu(0)} = 0$ which in turn implies that $\nabla_\mu B^{\mu\nu(0)} = 0$. These terms vanish because of the fact that one has to take into account the boundary theory at some fixed value of the chemical potential μ .

The $S^{(2)}$ coefficient is provided by the following relations:

$$\begin{aligned} - S^{(2)} &= \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{1}{2} F_{\mu\nu}^{(1)} F^{\mu\nu(0)} + \frac{1}{4} F_{\mu\nu}^{(1)} F^{\mu\nu(1)} \right. \\ &\quad \left. + \frac{\alpha}{4} \left(F_{\mu\nu}^{(2)} B^{\mu\nu(0)} + F_{\mu\nu}^{(1)} B^{\mu\nu(1)} + F_{\mu\nu}^{(0)} B^{\mu\nu(2)} \right) \right] \\ &= \int_{\partial\mathcal{M}} d\Sigma_u \left(F^{u\nu(0)} A_\nu^{(2)} + \frac{1}{2} F^{u\nu(1)} A_\nu^{(1)} \right) |_{u \rightarrow 0} - \frac{3}{2} \int_{\mathcal{M}} d^4x \sqrt{-g} \frac{j^{\nu(1)}}{\tilde{\alpha}} A_\nu^{(1)} \\ &\quad + \int_{\partial\mathcal{M}} d\Sigma_u \frac{\alpha}{2} \left(B^{u\nu(0)} A_\nu^{(2)} + F^{u\nu(0)} B_\nu^{(2)} + F_{\mu\nu}^{(1)} B^{\mu\nu(1)} \right) |_{u \rightarrow 0}, \end{aligned} \quad (4.6)$$

where one has used the equations of motion for the first order, i.e., $\tilde{\alpha}\nabla_\mu F^{\mu\nu(1)} = j^{\nu(1)}$. The 'orthogonality condition' [53]

$$\int_{\mathcal{M}} d^4x \sqrt{-g} A_\mu^{(1)} j^{\mu(1)} = 0, \quad (4.7)$$

can be implemented and the second term in the third line vanishes. Consequently we arrive at the following expression:

$$-S_{on-shell} = \int_{\partial\mathcal{M}} d\Sigma_u \left[\frac{\alpha}{2} \left(F^{u\nu(0)} B_\nu + B^{u\nu(0)} A_\nu \right) + \frac{\epsilon^2}{2} A_\nu^{(1)} \left(F^{u\nu(1)} + \alpha B^{u\nu(1)} \right) \right] + \mathcal{O}(\epsilon_H^3). \quad (4.8)$$

The same arguments as applied in $S^{(1)}$ analysis, lead us to the conclusion that the first term on the right-hand side of equation (4.8) vanishes. In the next step we use the fact that $A_t^{(1)}(\mathbf{x}, 0) = 0$. Consequently, by virtue of the above the on-shell action is provided by the relation of the form

$$-S_{on-shell} = \frac{\epsilon^2}{2} \int d^3x \delta^{ab} \left(F_{ua}^{(1)} + \alpha B_{ua}^{(1)} \right) A_b^{(1)}|_{u \rightarrow 0} + \mathcal{O}(\epsilon_H^3). \quad (4.9)$$

Having in mind the relation for $\langle J_a \rangle^{DM}$ one has that

$$-S_{on-shell} = \frac{\epsilon^2}{2} \int d^3x \delta^{ab} \langle J_a \rangle^{DM} A_b^{(1)}|_{u \rightarrow 0} + \mathcal{O}(\epsilon_H^3). \quad (4.10)$$

Putting together all those results, we obtain that the expression describing free energy for the considered theory with *dark matter* sector in $(2+1)$ -dimensions. It yields

$$F = -\frac{\epsilon^2}{2} \int_{\mathcal{R}^2} d^2\mathbf{x} \delta^{ab} \langle J_a \rangle^{DM} A_b^{(1)}|_{u \rightarrow 0} = -\frac{\epsilon^2}{2} B_{c2} \int_{\mathcal{R}^2} d^2\mathbf{x} \Theta(\mathbf{x}) - \frac{\epsilon^2}{2} \int_{\mathcal{R}^2} d^2\mathbf{x} \beta_i(\alpha) A^{i(1)}, \quad (4.11)$$

where B_{c2} stands for 'the upper magnetic' $U(1)$ -gauge Maxwell field.

Let us suppose that we take into account some bounded region V of the two-dimensional hypersurface \mathcal{R}^2 in question. The free energy density may be written as

$$\mathcal{F} = \frac{F}{V} = -\frac{\epsilon^2}{2} B_{c2} \langle \Theta(\mathbf{x}) \rangle - \frac{\epsilon^2}{2} \langle \beta_i(\alpha) A^{i(1)} \rangle, \quad (4.12)$$

where $\langle \Theta(\mathbf{x}) \rangle$ and $\langle \beta_i(\alpha) A^{i(1)} \rangle$ denote the averages of $\Theta(\mathbf{x})$ and $\beta_i(\alpha) A^{i(1)}$ over the two-dimensional hypersurface \mathcal{R}^2 .

It is worth pointing out that the free energy density consists of two terms, first one is connected with the ordinary Maxwell field while the other one is bounded with *dark matter* sector. Because of the fact that \mathcal{F} is negative one can conclude that the considered system is more stable over the trivial configuration where there is no charge condensate, i.e., the value of the scalar field ψ is equal to zero. Moreover, the free energy is smaller for the *dark matter* sector, which means that the vortices create more easily in the presence of *dark matter* than without it. Perhaps this could be the answer why the *cosmic web* of tendrils (resembling vortices in the large cosmological scales) form first and then an ordinary matter condenses on them to form galaxies and clusters of galaxies.

5 Superconducting coherence length

We turn now to the analysis of the superconducting length and the influence of the *dark matter* sector on it. In our analysis we shall consider the following components of the Maxwell and scalar fields:

$$A_\mu = (\phi(u), 0, 0, A_y), \quad \psi = \psi(u). \quad (5.1)$$

By virtue of the above choice we can straightforwardly verify that the equations of motion imply

$$\partial_u^2 \phi(u) - \frac{2 r_+ \psi^2 \phi}{\tilde{\alpha} u^4 f(u)} = 0, \quad (5.2)$$

$$\partial_u \left(f(u) \partial_u \psi \right) - \frac{m^2 r_+^2}{u^4} \psi + \frac{\phi^2 r_+^2}{f(u) u^4} \psi - \frac{1}{u^4} A_y^2 \psi = 0. \quad (5.3)$$

It happens that the superconducting coherence length ξ can be connected with the correlation length of the order parameter in the momentum space [48]. χ emerges as the pole of the static correlation function of the order parameter in the Fourier space. Namely, one can write that

$$\langle \mathcal{O}(k) \mathcal{O}(-k) \rangle \sim \frac{1}{|k|^2 + \frac{1}{\xi^2}}. \quad (5.4)$$

As in [48] the pole of the static correlation function of a dual field operator can be found by solving the eigenvalue problem for the static perturbation subject to the condition that the wave number k of the corresponding bulk field is given by $1/\xi^2 = -k_*^2$, where k_* is a wave number allowed as eigenvalues [48]. The aforementioned condition gives us the pole of the static correlation function. Because of the fact that near the critical point, when $T \rightarrow T_c$, the coherence length is divergent, one ought to solve the eigenvalue problem being subject to the condition

$$\lim_{\epsilon_T \rightarrow 0} (-k^2) = -k_*^2 = \frac{1}{\xi^2} = 0, \quad (5.5)$$

where $\epsilon_T = (T_c - T)/T_c$ with the auxiliary demand that $|\epsilon_T| \ll 1$. In order to solve the equations of motion let us perform the perturbative analysis in the fluctuations of the fields under considerations given by

$$\phi = \varphi(u) + \epsilon_T A_t^{(1)}(u) + \mathcal{O}(\epsilon_T^2), \quad (5.6)$$

$$\psi = \sqrt{\epsilon_T} \psi_1(u) + \epsilon_T^{\frac{3}{2}} \psi_2(u) + \mathcal{O}(\epsilon_T^2). \quad (5.7)$$

In the leading order of the ϵ_T parameter, the underlying equations of motion can be cast in the forms as stated below. Namely, in the first order of ϵ_T -order we get

$$\partial_u^2 A_t^{(1)}(u) - \frac{2 r_+^2}{\tilde{\alpha} u^4 f(u)} \psi_1^2(u) \varphi(u) = 0. \quad (5.8)$$

On the other hand, at $\sqrt{\epsilon_T}$ -order they imply

$$\partial_u \left(f(u) \partial_u \psi_1(u) \right) - \frac{m^2 r_+^2}{u^4} \psi_1(u) + \frac{r_+^2 \varphi^2(u) \psi_1(u)}{f(u) u^4} - \frac{1}{u^4} A_y^2 \psi_1(u) = 0, \quad (5.9)$$

and at $\epsilon_T^{\frac{3}{2}}$ -order, one arrives at the relation as follows:

$$\partial_u \left(f(u) \partial_u \psi_2(u) \right) - \frac{m^2 r_+^2}{u^4} \psi_2(u) + \frac{2 r_+^2 \varphi(u) \psi_1(u) A_t^{(1)}(u)}{f(u) u^4} = 0. \quad (5.10)$$

In order to determine the coherence length, we shall elaborate fluctuations around the background fields $A_\mu(u)$ and $\psi(u)$. It turned out that static perturbations will be adequate for our purpose. Thus, we pay attention to the fluctuations with only one spatial direction along x -direction. The linear perturbations of gauge fields and scalar one, will yield

$$\delta A_\mu = \left[a_t(u, k) dt + a_x(u, k) dx + a_y(u, k) dy \right] e^{ikx}, \quad (5.11)$$

$$\delta \psi = \left[b(u, k) + i \bar{b}(u, k) \right] e^{ikx}. \quad (5.12)$$

The boundary conditions for the gauge and scalar fields should be imposed. Namely, let us suppose that at the black brane horizon ($u = 1$) and near of the AdS spacetime boundary ($u = 0$), the asymptotic behavior of the fields in question imply

$$a_t(u, k) |_{u \rightarrow 1} = 0, \quad (5.13)$$

$$b(u, k) |_{u \rightarrow 1} = \text{regular}, \quad (5.14)$$

$$a_t(u, k) |_{u \rightarrow 0} = \text{const } u, \quad (5.15)$$

$$b(u, k) |_{u \rightarrow 0} = \text{const } u^2. \quad (5.16)$$

Intersecting (5.11)-(5.12) into the underlying equations of motion provides the following:

$$k^2 a_t = f(u) u^2 \partial_u^2 a_t - \frac{2 \psi^2 r_+^2 a_t}{\tilde{\alpha} u^2} - \frac{4 \psi b r_+}{\tilde{\alpha} u^2} \phi = 0, \quad (5.17)$$

$$k^2 a_y = \partial_u \left[u^2 f(u) \partial_u a_y \right] - \frac{2 \psi^2 r_+^2 a_y}{\tilde{\alpha} u^2} - \frac{4 \psi b A_y r_+^2}{\tilde{\alpha} u^2} = 0, \quad (5.18)$$

$$k^2 b = u^2 \partial_u \left[f(u) \partial_u b \right] + \frac{r_+^2 b \phi^2}{f(u) u^2} - A_y^2 b - \frac{m^2 r_+^2 b}{u^2} + \frac{2 a_t \phi \psi r_+^2}{f(u) u^2} - 2 a_t A_y \psi = 0, \quad (5.19)$$

$$\partial_u \left[u^2 f(u) \partial_u a_x \right] - \frac{2 r_+^2 \psi^2 a_x}{\tilde{\alpha} u^2} = 0. \quad (5.20)$$

The Wick rotation of the wave vector k , $k \rightarrow ik$ and substitution the relation (5.6) reveal the following:

$$-k^2 a_t = f(u) u^2 \partial_u^2 a_t - \frac{2 \epsilon_T \psi_1^2 r_+^2 a_t}{\tilde{\alpha} u^2} - \frac{4 \sqrt{\epsilon_T} \psi_1 b r_+ \varphi}{\tilde{\alpha} u^2} \phi = 0, \quad (5.21)$$

$$-k^2 b = \left(D_b + \frac{2 \epsilon_T r_+^2 \varphi A_t^{(1)}(u)}{f(u) u^2} \right) b + \frac{2 \sqrt{\epsilon_T} a_t \varphi \psi_1 r_+^2}{f(u) u^2} - 2 \sqrt{\epsilon_T} a_y A_y \psi_1 = 0, \quad (5.22)$$

where we have define the differential operator D_b by the relation

$$D_b = u^2 \partial_u \left(f(u) \partial_u \right) + \frac{r_+^2 \varphi^2}{f(u) u^2} - A_y^2 - \frac{m^2 r_+^2}{u^2}. \quad (5.23)$$

The zeroth order solutions of the set of equations (5.21)-(5.22), which are consistent with the asymptotical behavior near the black brane event horizon and AdS spacetime boundary, have the forms given by the relations

$$b^{(0)} = \psi_1, \quad a_t^{(0)} = 0, \quad a_y^{(0)} = 0. \quad (5.24)$$

The zeroth order solutions are in accord with the fact that at this order they are only functions of u -coordinate. Having in mind that perturbations described by the equations (5.11)-(5.12) have the spatial dependence on x -direction, we conclude that the first non-trivial corrections emerges at $\sqrt{\epsilon_T}$ -order. Thus, we suppose that

$$a_t = \sqrt{\epsilon_T} \tilde{a}_t, \quad a_y = \sqrt{\epsilon_T} \tilde{a}_y, \quad (5.25)$$

and substitute the above expressions into the relations for a_t , a_y , b . Consequently, one obtains

$$-k^2 \tilde{a}_t = f(u) u^2 \partial_u^2 \tilde{a}_t - \frac{4 \psi_1 b r_+^2 \varphi}{\tilde{\alpha} u^2} - 2 \epsilon_T \frac{\psi_1^2 r_+^2}{\tilde{\alpha} u^2} \tilde{a}_t = 0, \quad (5.26)$$

$$-k^2 b = D_b b + \epsilon_T \left(\frac{2 r_+^2 \varphi A_t^{(1)}(u) b}{f(u) u^2} + \frac{2 \tilde{a}_t \varphi \psi_1 r_+^2}{f(u) u^2} - 2 \tilde{a}_y A_y \psi_1 \right) = 0, \quad (5.27)$$

$$-k^2 \tilde{a}_y = \partial_u \left(u^2 f(u) \partial_u \tilde{a}_y \right) - \frac{4 \psi_1^2 A_y r_+^2}{\tilde{\alpha} u^2} + \mathcal{O}(\epsilon_T^{n \geq 1/2}). \quad (5.28)$$

Moreover we write down the set of expansions for the coefficients of the static perturbations

$$b = \psi_1 + \epsilon_T b^{(1)} + \mathcal{O}(\epsilon_T^2), \quad (5.29)$$

$$\tilde{a}_t = \tilde{a}_t^{(0)} + \mathcal{O}(\epsilon_T), \quad (5.30)$$

$$\tilde{a}_y = \tilde{a}_y^{(0)} + \mathcal{O}(\epsilon_T), \quad (5.31)$$

$$k_*^2 = \epsilon_T k_1^2 + \mathcal{O}(\epsilon_T^2). \quad (5.32)$$

By virtue of the equations (5.26)-(5.27) as well as (5.8)-(5.9), taking into account the limit $\epsilon_T \rightarrow 0$, which means that $k^2 \rightarrow k_*^2$, we can readily verify that

$$\partial_u^2 \tilde{a}_t^{(0)}(u, k_*) = \frac{4 \psi_1^2 r_+^2 \varphi}{\tilde{\alpha} f(u) u^4} = 2 \partial_u^2 A_t^{(1)}(u), \quad (5.33)$$

$$\begin{aligned} -k_1^2 \psi_1 &= D_b b^{(1)}(u, k_*) \\ &+ \left(\frac{2 r_+^2 \psi_1 \varphi A_t^{(1)}(u)}{f(u) u^2} + \frac{2 \tilde{a}_t^{(0)} \varphi \psi_1 r_+^2}{f(u) u^2} - 2 \tilde{a}_y^{(0)} A_y \psi_1 \right) = 0. \end{aligned} \quad (5.34)$$

In order to calculate the superconducting coherence length we define the scalar product defined by

$$\langle \psi_1 | \psi_1 \rangle = \int_0^1 du \frac{\psi_1^* \psi_1}{u^2}. \quad (5.35)$$

The form of the scalar product reveals the fact that the operator D_b is the Hermitian one. In the next step, we multiply the equation for \tilde{k}_1 by the bra $\langle \psi_1 |$. On this account it is customary to write

$$-k_1^2 \langle \psi_1 | \psi_1 \rangle = \langle \psi_1 | D_b b^{(1)} \rangle + \langle \psi_1 | \frac{2 r_+^2 \psi_1 \varphi A_t^{(1)}(u)}{f(u) u^2} \rangle - \frac{\tilde{\alpha}}{2} \int_0^1 du \left(\frac{d\tilde{a}_t^{(0)}}{du} \right)^2 - 2 A_y \int_0^1 \frac{du}{u^2} \psi_1^2 \tilde{a}_y^{(0)}. \quad (5.36)$$

The first term on the right-hand side can be found considering $\epsilon_T^{3/2}$ -order behavior of the equations of motion for the underlying theory. Namely the relation (5.10) enables us to write

$$D_b \psi_2 = -\frac{2 r_+^2 \psi_1 \varphi A_t^{(1)}(u)}{f(u) u^2}, \quad (5.37)$$

which in turn leads us to the conclusion that $\langle \psi_1 | D_b \psi_2 \rangle = 0$.

Finally, coming back to the original vector wave, by performing the Wick rotation as well as taking into account the limit when $\epsilon_T \rightarrow 0$, we receive the searched for expression

$$-k_1^2 = \frac{\mathcal{P}}{\mathcal{M}}. \quad (5.38)$$

For the brevity of the notation we have defined the above quantities \mathcal{P} and \mathcal{M} as

$$\mathcal{P} = \frac{\tilde{\alpha}}{2} \int_0^1 du \left(\frac{d\tilde{a}_t^{(0)}}{du} \right)^2 + 2 A_y \int_0^1 du \psi_1^2 \tilde{a}_y^{(0)}, \quad (5.39)$$

$$\mathcal{M} = \int_0^1 du \frac{\psi_1^* \psi_1}{u^2}. \quad (5.40)$$

On this account the superconducting coherence length in the theory with *dark matter* sector yields

$$\xi = \sqrt{\frac{\mathcal{M}}{\mathcal{P}}} \left(1 - \frac{T}{T_c} \right)^{\frac{1}{2}} \sim \tilde{\alpha}^{-\frac{1}{2}} \left(1 - \frac{T}{T_c} \right)^{\frac{1}{2}}. \quad (5.41)$$

The coherence length depends on the α -coupling constant (which in turn describes the dependence on the *dark matter* model). The main conclusion of these calculations is that the smaller α we consider, the greater value of the superconducting length we obtain.

6 London equation with *dark matter* sector

This section is devoted to calculations of the magnetic penetration depth and the number density of the superfluid for Maxwell *dark sector* vortices in $(2+1)$ -dimensions in the presence of a homogeneous external magnetic field. The magnetic field is chosen as perpendicular to the two-dimensional hypersurface. Let us introduce the ansatz of the form as

$$\delta A_y(u, x) = a_y(u) x, \quad (6.1)$$

which is equivalent to the perturbation of the gauge field with the wave vector $k = 0$. As was mentioned in section 2, A_y can be connected with the *dark matter* magnetic field or Maxwell magnetic field. For generality of our considerations we set in what follows A_y . Inserting it into equation (5.18) one readily writes down

$$\partial_u \left(u^2 f(u) \partial_u a_y \right) - \frac{2 r_+^2 \psi^2 a_y}{\tilde{\alpha} u^2} - \frac{4 \psi b A_y r_+^2}{\tilde{\alpha} u^2} = 0. \quad (6.2)$$

As in the previous section we assume the following perturbative expansion:

$$\psi = \sqrt{\epsilon_T} \psi_1(u) + \epsilon_T^{3/2} \psi_2(u) + \mathcal{O}(\epsilon_T^{5/2}), \quad (6.3)$$

$$a_y = a_y^{(0)} + \epsilon_T a_y^{(1)} + \mathcal{O}(\epsilon_T^3), \quad (6.4)$$

$$b = \psi_1(u) + \sqrt{\epsilon_T} b^{(1)} + \mathcal{O}(\epsilon_T). \quad (6.5)$$

Up to ϵ_T^0 and to ϵ_T^1 orders the equations are given respectively by

$$\partial_u \left(u^2 f(u) \partial_u a_y^{(0)} \right) = 0, \quad (6.6)$$

$$\partial_u \left(u^2 f(u) \partial_u a_y^{(1)} \right) - \frac{2 r_+^2 \psi_1^2(u) a_y^{(0)}}{\tilde{\alpha} u^2} - \frac{4 b^{(1)} \psi_1(u) r_+^2}{\tilde{\alpha} u^2} A_y = 0. \quad (6.7)$$

From the equation (6.6) it can be seen that $a_y^{(0)}$ is constant and $a_y^{(1)}$ solution can be formally written as

$$a_y^{(0)} = C_0 = \text{const}, \quad (6.8)$$

$$a_y^{(1)} = C_1 - 2 C_0 \int_0^u \frac{du''}{1 - u''^3} \int_{u''}^1 \frac{du'}{\tilde{\alpha} u'^2} \psi_1(u') \left(\psi_1(u') + \frac{2 b^{(1)}(u') A_y}{C_0} \right) + \mathcal{O}(\epsilon_T^2). \quad (6.9)$$

To proceed further, we set $C_1 = 0$ and $C_0 = A_y$. On this account the relation (6.1) for $\delta A_y(u, x)$ implies

$$\delta A_y(u, x) \simeq \delta A_y^{(0)}(x) \left(1 - 2 \epsilon_T u \int_{u''}^1 \frac{du'}{\tilde{\alpha} u'^2} \psi_1(u') \left(\psi_1(u') + 2 b^{(1)}(u') \right) \right) + \mathcal{O}(\epsilon_T^2), \quad (6.10)$$

where $\delta A_y^{(0)} = \lim_{u \rightarrow 0} \delta A_y(U, x) = A_y$.

On the other hand, from the asymptotic behavior of δA_y near the AdS spacetime boundary

$$\delta A_y = \delta A_y^{(0)} + \frac{\delta A_y^{(1)}}{r} + \dots, \quad (6.11)$$

and taking into account the definition $r_+ = 4\pi T_c/3$, we can read off the form of $\langle J_y(x) \rangle$ when $T \rightarrow T_c$. Namely, it is given by the relation

$$\langle J_y(x) \rangle_{u \rightarrow 0} = -\frac{8\pi \epsilon_T}{3} T_c \delta A_y^{(0)}(x) \int_0^1 \frac{du}{\tilde{\alpha} u^2} \mathcal{K}(u, k) + \mathcal{O}(\epsilon_T^2), \quad (6.12)$$

where we have denoted by $\mathcal{K}(u, k)$ the relation provided by

$$\mathcal{K}(u, k) = \psi_1^2(u) + 2 \psi_1(u) b^{(1)}. \quad (6.13)$$

Let us find the behavior of the integral (6.12) near the AdS spacetime boundary. Near the boundary (for $m^2 = -2$) the scalar field ψ_1 is proportional to the condensation operator $\langle \mathcal{O} \rangle$. Because of the fact that we reach almost the critical temperature, the condensation operator plays the role of the order parameter for the boundary theory. In general, the considered scalar field solution can be expressed as a function of the u -coordinate [48] in the form as

$$\psi(u) = \psi^m f(u), \quad (6.14)$$

where $f(u)$ is the solution of equation satisfying the limit $\lim_{u \rightarrow 0} f(u) = u^{\Delta_m}$, and regularity condition at the black hole event horizon. By virtue of this, we set

$$\psi_1 \sim \langle \mathcal{O} \rangle u^2. \quad (6.15)$$

For the upper limit in the considered integral, i.e., for $u = 1$, ψ_1 is perfectly regular. All these facts suggest that the integral in question is well defined and has the finite value.

As was revealed in [53] the form of $\langle J_y \rangle$ closely resembles the expression from the Ginzburg-Landau theory, so-called London equation, in which the order parameter ψ is coupled to the $U(1)$ -gauge field A_a and the current J_a is provided by

$$J_a = -\frac{e^2}{m} \psi^2 A_a = -e n_s A_a, \quad (6.16)$$

where e and m are effective charge and mass of the order parameter. n_s is connected with the superfluid number density. In the expression for the average spatial current in y -direction, $\delta A_y^{(0)}$ plays the role of an external source. On the contrary, in London equation A_a is built from the spatial average of the microscopic field as well as an external field. In the considered attitude we have no dynamical photon (the current does not produce its own magnetic field). Summing it all up, it means that the external $U(1)$ -gauge field $\delta A_y^{(0)}$ equals to the macroscopic gauge field in the AdS/CFT attitude. So the comparison of J_a and $\langle J_y \rangle$ reveals that, near the boundary of the AdS spacetime we have that the number density of the superfluid particles is given by

$$n_s^{DM} = \frac{8\pi T_c}{3 \tilde{\alpha}} \langle \mathcal{O} \rangle^2 F(u, k) = \frac{n_s}{\tilde{\alpha}}, \quad (6.17)$$

where $F(u, k)$ stands for the integral

$$F(u, k) = \int_0^1 \frac{du}{u^2} \mathcal{K}(u, k). \quad (6.18)$$

On the other hand, the magnetic penetration depth implies

$$\lambda^{DM} = \sqrt{\tilde{\alpha}} \lambda. \quad (6.19)$$

Inspection of the equations (6.17) and (6.19) reveals that the bigger α -coupling constant one takes the greater n_s^{DM} we get. On the contrary, the smaller α the greater magnetic penetration we have.

It is interesting to note that *dark matter* affects both the penetration depth and coherence length in the same way leaving their ratio intact $\lambda^{DM}/\xi^{DM} = \lambda/\xi$. Noting that this ratio decides if the superconductor is of first or second type, the conclusion is that, at least in the probe limit, the dark matter does not affect classification of the holographic superconductors.

7 Conclusions

In the paper we have considered AdS/CFT - gauge/gravity, correspondence in order to study linear fluctuations of scalar field solution in the s-wave holographic superconductor under the assumption of the probe limit, i.e., the fluctuations do not backreact on the gravitational field. We have analyzed the theory in which $U(1)$ -gauge Maxwell field is coupled with the other gauge field which mimics the presence of the *dark matter* sector.

The main aim of the work was to answer the question how *dark matter* sector (α coupling constant of these aforementioned gauge fields) will modify characteristics of vortex solutions, like the coherence length, superfluid density and magnetic penetration depth. It happened that both, the coherence length and the penetration depth were affected by the presence of the *dark matter* sector in the same way. The smaller value of α is taken into account the greater value of the ξ and λ is obtained. On the other hand, the smaller α we have, the greater superfluid density we get.

We have also found that the free energy for the vortex configuration turns out to be negative and for the configuration with *dark matter* sector lesser than in the 'ordinary' Maxwell case. This fact enables us to conclude that the vortex configurations are stable over the trivial configuration where there is no charge scalar condensate. Secondly, because of the fact that in the presence of *dark matter* the vortices build less free energy configuration than ordinary matter ones (e.g., Maxwell or Chern-Simons), it can give rise to the answer why in the Early Universe first the web of *dark matter* formed and next on these tendrils the ordinary matter condensed forming cluster of galaxies.

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